Review for Final – Financial Math

Finite Math

8 May 2017

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where $A =$ *future value,* $P =$*principal/present value,* $r =$*annual simple interest rate,* $t =$*time in years.*

Example

If an investor wants to earn an annual interest rate of 6.4% *on a* 26*-week T-bill with a maturity value of* $\$5,000$*, how much should the investor pay for the T-bill*

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- $n =$ *total number of compounding periods*

Example

If $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Example

How long will it take $\$4,000$ *to grow to* $\$10,000$ *if it is invested at* 6% *compounded monthly?* 9% *compounded monthly?*

Continuous Compound Interest

Definition (Continuous Compound Interest)

Principal P invested at an annual nominal rate r will have future value

$$A = Pe^{rt}$$

after time t (in years).

Example

If you invest $\$5,650$ *in an account paying* 8.65% *compounded continuously, how much money will be in the account at the end of* 10 *years?*

Example

A couple wishes to have $\$40,000$ *in* 6 *years for the down payment on a house. If the couple has* $\$25,000$ *to invest, what interest rate (a) compounded quarterly, (b) compounded monthly, (c) compounded continuously should the couple look for?*

APY

Definition (Annual Percentage Yield)

If a principal is invested at the annual (nominal) rate r compounded m times a year, then the annual percentage yield is

$$APY = \left(1 + \frac{r}{m}\right)^m - 1$$

If a principal is invested at the annual (nominal) rate r compounded continuously, then the annual percentage yield is

$$APY = e^r - 1$$

Example

Which is the better investment and why: 9% *compounded quarterly or* 9.25% *compounded annually?*

Future Value/Sinking Fund

Definition (Future Value of an Ordinary Annuity)

$$FV = PMT\frac{\left(1+\frac{r}{m}\right)^{n}-1}{r/m}$$

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Note that the payments are made at the end of each period.

Example

A company establishes a sinking fund for plant retooling in 6 *years at an estimated cost of* $\$850,000$. *How much should be invested semiannually into an account paying* 8.76% *compounded semiannually?*

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Example

A couple has a $\$50,000$*, 10-year mortgage at* 9% *compounded monthly. What will their monthly payment be?*

Example

A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 20 *years so that he can then make quarterly withdrawals of* $\$5,000$ *for* 10 *years. The annuity earns* 7.32% *interest compounded quarterly.*

(a) *How much will have to be in the account at the time he retires?*

(b) *How much should be deposited each quarter for* 20 *years in order to accumulate the required amount?*

(c) *What is the total amount of interest earned during the* 30*-year period?*